Indian J. Phys. **77B** (3), 283-289 (2003)

IJP B

Multipath environment indoor analysis by Sommerfeld integrals, geometric optics and Norton surface-wave

R Linares y Miranda*, J L López-Bonilla*, M Acevedo and R Meneses
Posgradute Section of the Superior School of Mechanical and Electrical Engineering, Politechnic Institute of Mexico Zacatenco Campus, U P A L M., Edif Z. Acc 3, 3er Piso, Col. Lindavista, C P 07738, México, D F

E-mail rlinares@maya esimez.ipn mx

Received 27 June 2002, accepted 18 February 2003

tract In the analysis of indoor wireless communication systems the multipath environment is a concerning topic and optimum models are ed for A frequency and time-domain investigation for an indoor environment by Sommerfeld Integrals, Geometric Optics and Norton Surface- in relation to an elementary dipole radiation above a ground plane (plane of reference) has been presented in US Department of Commerce, NTIA ort 00-379 (200) In this work, these models were applied using a two-elements antenna and the probabilistic model [*IEEE Trans Antennas agat* 48 1161 (2000)] of a multipath indoor analysis is presented. The results show Sommerfeld integrals have more accuracy of 6 DB than metric Optics plus Norton Surface-Wave and of 11 dB than Geometric Optics

words Multipath environment, indoor propagation

S Nos. 41 20 -q, 03.50.De, 41 60.-m

Introduction

ultipath electromagnetic environment modeling is very mplex and assumptions have been made in order to obtain timum models. One of the multipath effects is the null in liation pattern of the receiver antenna. In an effort to mitigate th effects, several studies proponing new antennas have been ade [1]-[3]. Based on the concept of dominant path, Young *et* [3] presented a dipole loop-capacitor combination antenna, hich exploits the phase shift produced by the capacitor between e dipole and loop currents for null suppression. Various proaches have been presented as well to analyze the multipath nvironment with different techniques. The Ray tracing chnique [4, 5] is the most popular to determine impulse sponse delay spread. This technique is based on Geometric tics (GO) and assumes that Fresnel reflection coefficients are lid for indoor applications, where near surface and near field are negligible. Therefore, the GO approach for indoor applications is in question, since the reflecting planes are close to the receptor antenna. Cotton *et al.* [6] presented an analysis with regard to the shortcomings of the GO approximations for antennas close to the reflecting planes and concluded that the GO approach is inadequate.

Problems relative to optimum models for multipath analysis are numerical approximations, which are focused on the classic problem of dipole radiation above a lossy half space. The numerical analysis of dipole antennas near a dielectric and a conductor half-space has been approached in various ways. Cotton *et al.* [6] evaluated Sommerfeld integrals and compared them to corresponding Geometric Optics approximations. Michalski [7] used a numerically integrated mixed-potential Green's functions formulation to model an oblique half-wave dipole antenna as a function of angle. Lindell *et al.* [8] presented a numerical analysis with continuous image source intensity for Green's function and induced electromagnetic force method to calculate the impedance of a horizontal dipole. Burke and Miller [9] precalculated the Sommerfeld integrals using interpolation to apply the moments method for modeling of currents on thin wires; this approach is used in the Numerical Electromagnetic Code (NEC) modeling half wave dipole input resistance as a

responding Author
ress for correspondence :
Superior de Ingenieria Mecánica y Eléctrica Unidad Zacatenco dad Profesional Adolfo López Mateos Edif. Z, Acceso 3, 3er Piso, Lindavista C P. 07738, México, D.F.

function of height above a lossy half space. Popovic and Djurdjevic [10] used an entire domain polynomial basis function and an unspecified compact algorithm to evaluate the Sommerfeld integral.

We undertake the problem by analyzing the scenario with a two element antenna, which reduces the probability of a deep null in the received signal for the few centimeters of distance between the antenna and the reflecting plane. We start with the statement of the problem summarizing the classical formulation of an antenna near an infinite plane conductor and provide the Sommerfeld formulation and Geometric Optics expression for an elementary dipole and loop above an infinite ground plane. Then, numerical techniques are given to evaluate the Sommerfeld integrals and finally, the results and details of the approach for a dipole loop-capacitor antenna, our goal of multipath analysis has been described

2. Statement of the problem

The multipath electromagnetic environment is analyzed according to the accuracy of analytical models. It assumes isotropic antennas. The accuracy is given in terms of complex solutions of field equations that govern a determined antenna Cotton *et al* [6] presented the formulation for an electrical dipole above a ground plane with a Sommerfeld Integral (SI), Geometric Optics (GO), and Norton Surface-Wave (NSW). Cotton made a comparison of such models in accordance with the accuracy in frequency and time domain for an indoor application. Young *et al* [3] presented a two-element antenna for null suppression in a multipath environment. With this antenna, the probability of a deep null in the received signal has been reduced in comparison to a dipole antenna. The two-element antenna consists of a dipole terminated with a parallel loop capacitor combination which exploits the shift phase between the dipole and loop currents to probabilistically reduce the deep null. The development of this antenna is simple and with the most accurate formulation as that presented by Cotton, the analysis of a multipath environment may be made.

In a multipath environment, the two-element antenna responds to the incident and reflected plane waves (E_u and H_u) when the dipole is aligned with the a-axis, which yields an output voltage that may be obtained by

$$V = C\,E_a + \eta_0 H_a e^{j(\gamma+\pi/2)} \qquad (1)$$

where C is a constant gain having units of length, η_0 is the impedance of free-space, γ is the extra phase introduced between the loop and dipole by the capacitor, E_a is the electric field component of the incident wave in the dipole antenna and H_a is the magnetic field component of the incident wave in the loop antenna

Null probabilistic analysis may be done with the following approach [3]

$$f_v(\nu) = \begin{cases} \dfrac{2\nu}{\pi^2\sqrt{1-\cos^2\gamma}}\, K\left(\sqrt{\dfrac{\nu^2\left(1-(\nu/2)^2\right)}{1-\cos^2\gamma}}\right) & 0<\nu<\sqrt{2(1-\cos\gamma)} \\ +\infty, & \nu=\sqrt{2(1-\cos\gamma)} \\ \dfrac{2\nu}{\pi^2\sqrt{\nu^2\left(1-(\nu/2)^2\right)}}\, K\left(\sqrt{\dfrac{1-\cos^2\gamma}{\nu^2\left(1-(\nu/2)^2\right)}}\right) & \sqrt{2(1-\cos\gamma)}<\nu<\sqrt{2(1+\cos\gamma)} \\ +\infty & \nu=\sqrt{2(1+\cos\gamma)} \\ \dfrac{2\nu}{\pi^2\sqrt{1-\cos^2\gamma}}\, K\left(\sqrt{\dfrac{\nu^2\left(1-(\nu/2)^2\right)}{1-\cos^2\gamma}}\right) & \sqrt{2(1+\cos\gamma)}<\nu<2 \end{cases} \qquad (2)$$

where $K(.)$ denotes the complete elliptic integral, *i.e*

$$K(k) = \int_0^1 \left[\left(1-x^2\right)\left(1-k^2x^2\right)\right]^{-1/2} dx .$$

With the objective of comparison an antenna dipole has

$$f_{v_{dipole}}(\nu) = \frac{2}{\pi^2} K\left(\sqrt{1-(\nu/2)^2}\right) \qquad 0<\nu<2 .$$

Then, by obtaining the field components, an analysis of the electromagnetic environment for dominant paths can be carried out.

Electromagnetic field components of incident wave pertaining to the two-element antenna may be determined by means of the formulation that is presented in the following section.

3. Formulation for field components received by a t antenna

In order to analyze the behavior of a multipath environment using a two-element antenna, the classic problem of image theory for an elementary dipole radiation above a ground plane is introduced [11-13]. The geometry is shown in Figure 1, following typical nomenclature. Region 0 is free space and region 1 (ground plane) a material whose composition is defined by relative dielectric constant ε_r and its conductivity σ. In region 0, a radiation source and observation points are placed. The distance of the source above the interface ground plane free space is h_T and the distance of the observation point above the interface ground plane free space is h_R. The source and the ıbser\ ın point are separated by a distance ρ , without :onsid ıg nature and orientation. Let us assume that the .ource iıates in all directions and at the observation points, here :. lirect and reflected rays. The direction is determined ıy the v's of reflection which assures that the energy in a ıomoį .ous media travels in straight lines along the shortest ıath

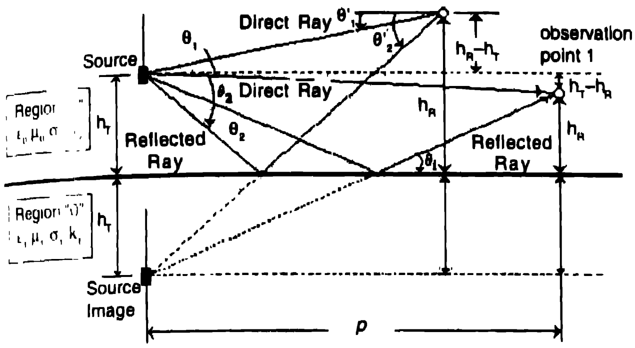

Figure 1. Geometry for electric field components above a dielectric half-space

For the geometry of two media where one is free space and the other can be a dielectric or a conductor, the Helmholtz equation in both media must be satisfied as the boundary condition applied at the interface.

The fields' evaluation above the ground plane may be carried out by means of a Geometric Optics (GO) approximation, a Norton Surface-Wave (NSW) approximation, or Sommerfeld Integrals (SI). GO is normally used in far-fields that assume there are several wavelengths in the distance between the source, the observation point, and the reflection plane. A NSW approximation is applied for large distances and a SI is used when the interaction distance is small.

It has been found that a Sommerfeld representation is in fact the most adequate solution for the fields of current elements in the presence of a ground plane, but it contains integrals that are complex. This representation is obtained from a double Fourier integral by transforming to cylindrical coordinates that introduce a Bessel function of zero order to the integral solution. Numerical schemes for evaluating these integrals have been developed [11, 12] but they all have limited applicability. The most accurate solution for effects of a ground plane use the Sommerfeld formulation for interaction of distances less than one wavelength and asymptotic expansion for larger distances [13], other solutions use extrapolation methods to accelerate the convergence of such integrals [14].

The c· nplete formulation for a Vertical Electric Dipole and a Horizont· Electric Dipole are given in [6, 11, 13]. Here, only the field co· ›onent expressions for analysis of multipath environn· ·ts in accordance with a two-element antenna are presente·

3.1. Sommerfeld representation :

$$E_z^V = A_d \left| \left| \frac{\partial}{\partial z^2} + k_0^2 \right| (G_0 + G_1 2Q) \right|, \quad A_d = \frac{j\omega p \mu_0}{4\pi k_0^2} \tag{4}$$

$$H_z^H = -\frac{p}{2\pi}\sin(\phi)\left[\frac{\partial}{\partial \rho}(G_0 - G_1 + U)\right], \tag{5}$$

where p is a conventional electric dipole moment. In order to have an ideal communication link, it assumes that transmission signals have a flat response at spectrum frequency. Thus, the electric dipole moment can be defined as $p = j\frac{4\pi}{\omega\mu_0}$. Besides, G_0 and G_1 are the free space (region 0) and image (region 1) Green's functions. The sum ($G_0 + G_1$) of the two represents the condition when region 1 is a perfect conductor and interface between the two media $G_0 = G_1$. These function are given by

$$G_0 = (1/R_0)e^{-jk_0R_0} = \int_{-\infty}^{\infty} \frac{e^{-\ell|h_T - h_R|}}{\ell} J_0(\lambda\rho)\,\lambda d\lambda, \tag{6}$$

$$G_1 = (1/R_1)e^{-jk_0R_1} = \int^{\infty} \frac{e^{-\ell(h_T + h_R)}}{\ell} J_0(\lambda\rho)\,\lambda d\lambda, \tag{7}$$

where $J_0(\lambda\rho)$ is the Bessel function of the first kind of zero order; R_0 and R_1 are the distances from the observation point to the source and image respectively, and are given by

$$R_0 = \sqrt{\rho^2 + (h_R - h_T)^2}, \quad R_1 = \sqrt{\rho^2 + (h_R + h_T)^2}. \tag{8}$$

Q and U are Sommerfeld integrals

$$Q = \int_0^{\infty} \frac{mk_0^2}{\ell\left(\ell k_1^2 + mk_0^2\right)} e^{-\ell(h_R + h_T)} J_0(\lambda\rho)\,\lambda d\rho, \tag{9}$$

$$U = \int_0^{\infty} \frac{2}{(\ell + m)} e^{-\ell(h_R + h_T)} J_0(\lambda\rho)\,\lambda d\rho,$$

where $\ell = \sqrt{\lambda^2 + k_0^2}$ and $m = \sqrt{\lambda^2 - k_1^2}$. We note that Q represents [15] the correction for the loss characteristic of the region 1 material.

3.2. Geometric optics approximation :

$$E_z^V = -\frac{jk_0 p}{4\pi}\eta_0\left[\sin^2\theta_d G_0 + \Gamma_\parallel \sin^2\theta_r G_1\right], \tag{10}$$

$$H_z^H = -\frac{jk_0 p}{4\pi}\sin\phi\left[\sin\theta_d G_0 + \Gamma_\perp \sin\theta_r G_1\right], \tag{11}$$

where θ_d and θ_r correspond to angles of the direct and reflected rays which are shown in Figure 1 as θ_1 and θ_2, respectively.

3.3. Norton surface-wave approximation :

$$E_z^V = -\frac{jk_0P}{4\pi}\eta_0\left[(1-\Gamma_\parallel)F(w)\sin^2\theta_r G_1\right], \tag{12}$$

$$H_z^H = -\frac{jk_0P}{4\pi}\sin\phi\left[(1-\Gamma_\perp)F(q)\sin^2\theta_r G_1\right], \tag{13}$$

where

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}},$$

$$F(w) = 1-\sqrt{\pi w}\exp(-w)\,erfc(j\sqrt{w}),$$

$$w = -\frac{jk_0R_0}{2\sin^2\theta_r}(\cos\theta_r + \Delta_0)^2,$$

$$\Delta_0 = \frac{k_0}{k_1}\sqrt{1-\left(\frac{k_0}{k_1}\right)^2\sin^2\theta_r},$$

$$q = -\frac{jk_0R_0}{2\sin^2\theta_r}(\cos\theta_r + \delta_0)^2,$$

$$\delta_0 = \frac{k_1}{k_0}\sqrt{1-\left(\frac{k_1}{k_0}\right)^2\sin^2\theta_r},$$

$$\Gamma_\parallel = \frac{\kappa_\varepsilon\cos\theta_i - \sqrt{\kappa_\varepsilon - \sin^2\theta_i}}{\kappa_\varepsilon\cos\theta_i + \sqrt{\kappa_\varepsilon - \sin^2\theta_i}},$$

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{\kappa_\varepsilon - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\kappa_\varepsilon - \sin^2\theta_i}}$$

and $\kappa_\varepsilon = \left|\varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0}\right|$, that is the index of reflection coefficient.

$\Gamma_\parallel$ and $\Gamma_\perp$ represent the Fresnel vertical reflection coefficient and the Fresnel horizontal reflection coefficient, respectively.

In wave propagation analysis, the materials parameters are sometimes considered frequency-independent, but electric properties of building materials vary with frequency and their effects modify the reflection coefficients ($\Gamma_\parallel$ and $\Gamma_\perp$). In order to analyze such coefficients, their behavior as a function of frequency and incident angle are shown in for reinforced concrete (RC) (Figure 2) and for ceiling/floor (Figure 3) for average values of their dielectric constants and conductivity

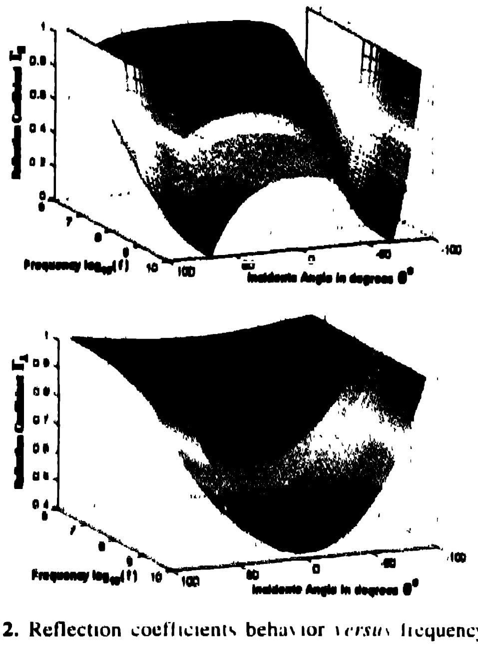

Figure 2. Reflection coefficients behavior *versus* frequency and incident angle for reinforced concrete (ε_r = 6.1, σ = 0.065 *S/m* [16])

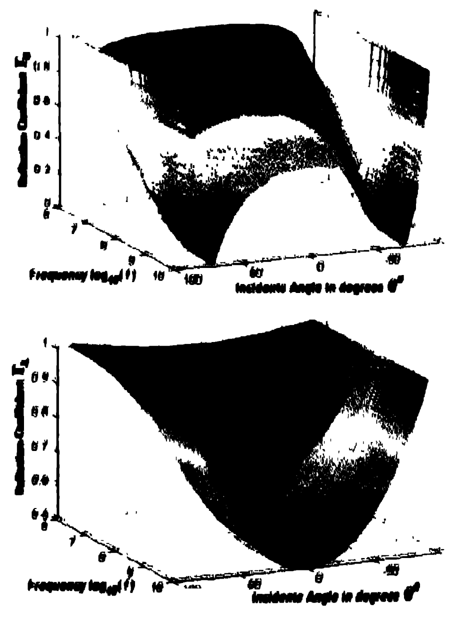

Figure 3. Reflection coefficients behavior *versus* frequency and incident angle for ceiling/floor (ε_r = 9, σ = 0.075 *S/m* [16])

4. Numerical techniques and evaluation

A Sommerfeld integral solution is obtained via numerical techniques [6-8, 14]. In this case the expressions (9) are evaluated by numerical integration along contours in the λ complex plane their poles may cause strong variations which are symmetrically located in the fourth quadrant on the plane complex. These integrands have branch cuts from $\pm k_0$ to infinity and $\pm k_1$ to infinity due to the square roots in ℓ and m respectively. The poles and branch points in the complex plane are shown in Figure 4. In order to avoid branch cut, the key for numerical evaluation is in the selection of the integration intervals. A rapid convergence in numerical integration exploits the exponential behavior and Bessel function for large λ .

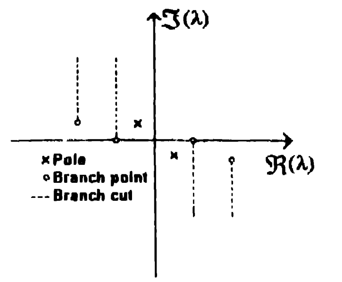

Figure 4. Sommerfeld poles and branch at complex plane λ

Three sub-intervals are selected to avoid branch cut, which are $[0, k_0]$, $\left[k_0, k_0\sqrt{\varepsilon_r}\right]$ and $\left[k_0\sqrt{\varepsilon_r}, \infty\right]$. For the first and second sub-interval, Romberg quadrature is applied due to it being an effective technique over a finite range of integration. Using the trapezoidal rule and by means of the associated extrapolation error a higher order approximation may be obtained [17]. Then for N times of equal intervals, the extended trapezoid can be applied *i.e.*

$$
S = \int X(\xi)d\xi = \Delta\xi\left|\frac{1}{2}X(\xi_0) + X(\xi_1)+\ldots+\frac{1}{2}X(\xi_N)\right)
$$

$$
\frac{B_2(\xi_N - \xi_0)^2}{2!N^2}\left[X^1(\xi_N) - X^1(\xi_0)\right]-\ldots \tag{14}
$$

$$
-\frac{B_{2n}(\xi_N - \xi_0)^{2n}}{(2n)!N^{2n}}\left[X^{(2n-1)}(\xi_N) - X^{(2n-1)}(\xi_0)\right]-\ldots
$$

where B_n is a Bernoulli number.

To evaluate first with N steps and next with $2N$ steps, the leading error in the second evaluation shall be 1/4 the size of error in the first evaluation. Then, the error for the two combinations is $S = \frac{4}{3}S_{2N} - \frac{1}{3}S_N$. Changing variable in the object, the discontinuity in the derivative at k_0 is removed and then, in the first sub-interval, the variables $\lambda = \xi = k_0 \cos v$ change to

$$
\int_0^{k_0} X(\xi)d\xi = \int^{\frac{\pi}{2}} X(k_0 \cos v)\, k_0 \sin v dv. \tag{15}
$$

In the second sub-interval, the variables $\lambda = \xi = k_0 \cosh \vartheta$ change to

$$
\int^{k_0\sqrt{\varepsilon_r}} X(\xi)d\xi = \int^{\mathrm{Arccosh}\sqrt{\varepsilon_r}} X(k_0 \cosh\vartheta)k_0 \sinh \vartheta d\vartheta. \tag{16}
$$

In the case of semi-infinite subintervals with integration functions that are slowly convergent like Sommerfeld integrals, the weighted-averages method is a useful numerical technique. This method is used to accelerate the convergence by extrapolation and it is a more sophisticated version of the Euler transformation, which is based on weighted means of the consecutive partial sum. The weights are selected on the remainder estimation. To derive this algorithm, one begins with determining the sequence of the partial sums by means of integration and then doing a summation procedure in which the integral is evaluated as a sum of a series of partial integrals over finite sub-intervals. If the weight (W_N) is associated with the sequence of a partial sum (S_N), the generalized weighted-averages algorithm may be expressed by

$$
S^{(l+1)} = \frac{S^{(l)} + \alpha_N^{(l)} S_{N+1}^{(l)}}{1+\alpha_N^{(l)}} \quad (N \geq 0 \text{ and } l \geq 0), \tag{17}
$$

where

$$
\alpha_N = \frac{W_{N+1}}{W_N} - \frac{r_N}{r_{N+1}}
$$ and r_N is the remainder estimate.

The parenthesized superscripts denote transformation order.

Now, a semi-infinite interval begin with $\lambda = \xi$ at $\xi_N = k_0\sqrt{\varepsilon_r}$ since all singularities, poles and branch points on the right half of the complex plane lie either on or near the line defined by $\Re(\xi) = k_0\sqrt{\varepsilon_r}$. The break points $\xi_N = k_0\sqrt{\varepsilon_r} + N\pi / \rho$ were chosen based on the half-period of the Bessel function, π / ρ and the weights were chosen according to the analytical form of the remainder estimates given by Michalski [14]

$$
W_N = \frac{(-1)^{N+1}}{\xi_N^{\beta-1/2}}\exp\left|-\frac{N\pi(h_R + h_T)}{\rho}\right| \tag{18}
$$

where β is the asymptotic coefficient. $\beta = 1$ for the electric field component and for the magnetic field component, $\beta = 0$, then

$$
\alpha_N^{(l)} = \exp\left[\frac{\pi(h_R + h_T)}{\rho}\right]\left(\frac{\xi_{N+1}}{\xi_N}\right)^{\beta-1/2+l} \tag{19}
$$

5. Frequency domain analysis

The evaluation appropriated to communication links must be carried out carefully, considering propagation effects. Several models have been proposed which are valid at a determined frequency interval. For the range of personal communication systems, the most popular model is Geometric Optics (GO) but other models may give more accuracy. The Norton Surface-Wave (NSW) model is used for longer distances and its analysis does not apply in such systems, but a combination of GO and NSW may give the maximum accuracy. Then, this combination and Sommerfeld Integrals can be compared to the two-element response given by eq. (1).

For frequency analysis, the main parameters to consider are: electric dipole moment, electric properties of the material of the loss half space, frequency range and geometry dimension. In order to have an ideal communication link, it is assumed that transmission signals have a flat response at spectrum frequency. Electrical properties of the material are defined by the conductivity and relative dielectric constant. In this case, typical arbitrary values of building material are chosen, such as $\varepsilon_r = 5$ and $\sigma = 10^{-3} - 10^{3}(S/m)$. For the analysis, a frequency range of $10^7 - 10^{10}\,Hz$ is chosen which covers wireless LAN currents and future applications such as yet-unlicensed bands (5.725-5.850 GHz). Geometry dimension is fixed to $\rho = 5m$, $h_T = h_R$ varying from 0.01 m to 10 m to cover near-field and far-field effects.

For a two-element antenna, an extra phase of $\gamma = \pi/4$ between dipole and loop is considered. The responses of such an antenna as a function of frequency, comparing the approximations mentioned (SI, GO and GO+NSW) are shown in Figures 5 and 6. These Figures show the two-element antenna behavior for different conductivities and two different dimensions respectively.

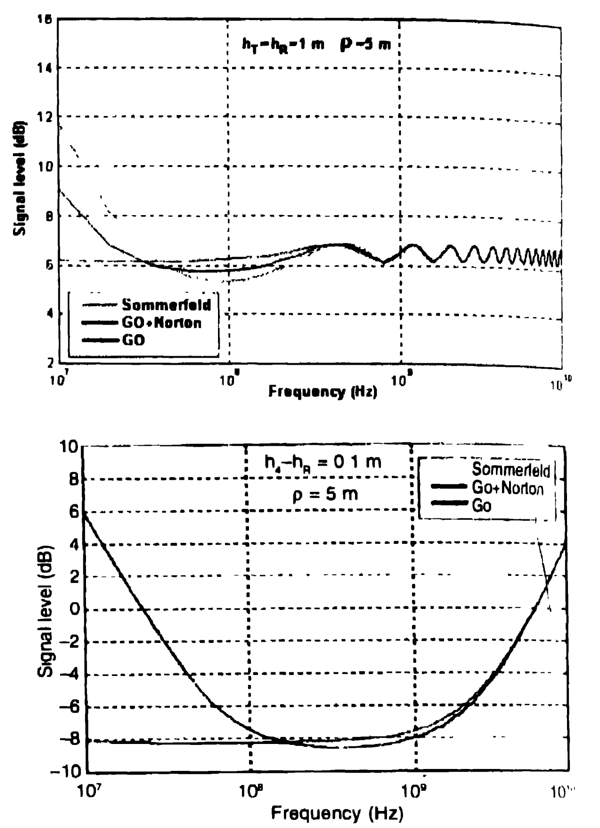

Figure 6. Level signal of a two-element antenna [3] for two different dimensions with $\varepsilon_r = 5$ and $\sigma = 10^{-3}$ (*S/m*)

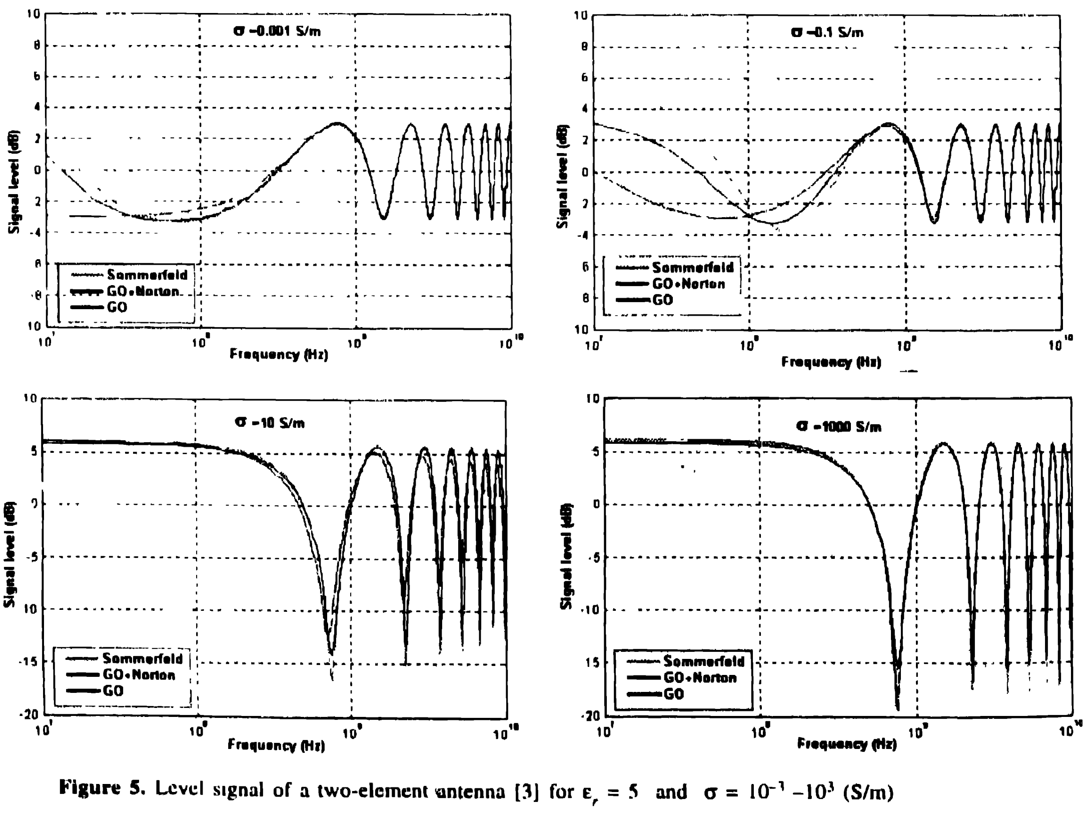

Figure 5. Level signal of a two-element antenna [3] for $\varepsilon_r = 5$ and $\sigma = 10^{-1} - 10^{3}$ (S/m)

The comparison of the evaluation SI, GO+NSW, and GO for $\sigma > 10$ (conductivity) showed that the three techniques have the same behaviour. But with small conductivities not equal to zero, differences are present and the least accurate is the GO approximation. These differences are more critical in $f > 10^9$ Hz. GO approximation is easier to apply, because it does not need special functions and numerical solutions.

6. Probabilistic analysis

In the indoor environment, the radio wave propagations are given *via* multi-reflector planes placed near and between them. In this environment, the radio transmitter/receptor is moved randomly, then the propagation analysis must be probabilistic. This section presents a comparison of the Cumulative Distribution Functions (CDF) of the two-element antenna [eq. 2)] in accordance with the propagation models mentioned. The data used are : frequency of 475 MHz, reflector plane with $\varepsilon_r = 5$ and $\sigma = 0.001$ S/m ; shift phase $\gamma = \pi/4$ between dipole and loop current, the wave polarization for the magnetic field component $\phi = 90^0$, the antennas (transmitter/receptor) height equals $h_T = h_R = 1m$, and the distance between the antennas is random $\rho = 1$ to 5 m. The probabilistic analysis is made in accordance with the simulation procedure given by Young *et al* [3] for SI, GO+NSW, and GO. The CDFs of the two-element antenna for voltages obtained from SI, GO+NSW, and GO with the specified data are shown in the Figure 7.

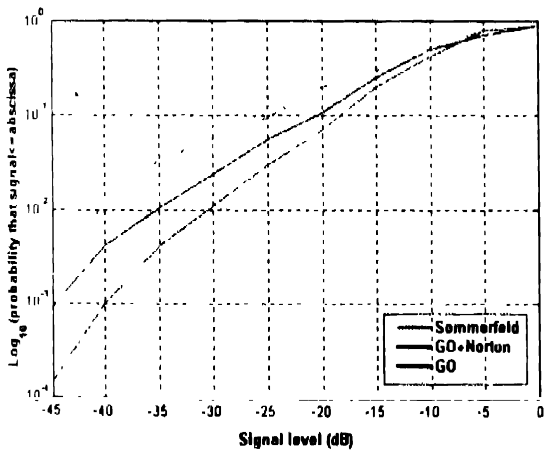

Figure 7. Two-element antenna CDFs for ρ = 0 01 to 1 m with different propagation models.

7. Conclusion

The analysis made in this work show that the propagation model obtained with the numerical solution of the Sommerfeld integrals is not accurate, while the Geometric Optics approximation gives least accuracy. In spite of the efficient computational tools, a statistical analysis using the Sommerfeld integration is complex, but a good option is the Geometric Optics plus Norton Surface-Wave approximation which gives less error than Geometric Optics. The difference between the Sommerfeld integration and the GO+Norton in the probabilistic analysis (10^{-2}) is 6 dB, while it is 11 dB for GO for signal levels of -30 dB.

Acknowledgments

We thank the Council of Science and Technology of Mexico (CONACyT). RL is grateful to Dr. Robert Olsen of Washington State University for kind assistance.

References

[1] M A Jensen and Y Rahmat-Samii *IEE Proc H* **141** 85 (1994)

[2] M Muramoto, N Ishii and K Itoh *IEEE Trans Antennas Propagat* **45** 1818 (1997)

[3] W F Young, B Belzer and R G Olsen *IEEE Trans Antennas Propagat* **48** 1161 (2000)

[4] G Durgin, N Patwari and T S Rappaport *Electronic Lett* **33** 1412 (1997)

[5] R P Torres, L Valle, M Domingo, S Loredo, and M C Diez *IEEE Antennas and Propagat Mag* **41** 11 (1999)

[6] M G Cotton, E F Kuester Christopher L Holloway *U S Department of Commerce* **NTIA Report 00-379** (2000)

[7] K A Michalski *Archil Elektron. Ubertragungstech* **39** 317 (1985)

[8] I V Lindell, E Alanen and K Mannersalo *IEEE Trans Antennas Propagat* **33** 937 (1985)

[9] G J Burke and E K Miller *IEEE Trans. Antennas Propagat* **32** 1040 (1984)

[10] B D Popovic and D Z Djurdjevic *Proc. Inst Elect. Eng Microwaves Antennas Propagat* **142** 213 (1995)

[11] A Baños *Dipole Radiation in the Presence of a Conducting Half-Space* (New York. Pergamon Press) (1966)

[12] E K Miller, A J Poggio, G J Burke and E S Selden *Analysis of Wire Antenna in Presence of a Conductor Half Space: Part I Vertical Antenna in Free Space. Part II*, **UCRL-73217** (Lawrence Livermore Laboratory, CA) (1972)

[13] *Numerical Electromagnetic Code (NEC-1) Part II· Program Description Code* (Lawrence Livermore Laboratory, CA) (1977)

[14] K A Michalski *IEEE Trans Antennas Propagat.* **46** 1405 (1998)

[15] B Van-der Pol *Physica* **2** 843 (1935)

[16] Chung-Fa Yang, Boau-Cheng Wu and Chuen-Jyi Ko *IEEE Trans. Antennas Propagat.* **48** 907 (1998)

[17] Peter V O' Neil Advanced *Engineering Mathematic* (Belmont, CA : Wadsworth) (1987)